Time Value of Money

Learning Problems

**Answer Keys**

**Problem: Future Value at Hamilton**

1. (100,000.00) (1 + .02)3 = 106,120.80 FV = CAD 106,120.80

106,120.80 – 100,000.00 = 6,120.80 Interest = CAD 6,120.80

**Note:** The original amount invested in the bank deposit is deducted from the future value to determine the amount of the interest earned.

1. (100,000) (1 + .02)5 = 110,408.08 FV = CAD 110,408.08

110,408.08 – 100,000 = 10,408.08 Interest = CAD 10,408.08

**Note:** More interest is earned because the term of the investment is longer.

1. (100,000) (1 + )(12 x 3) = 106,178.35 FV = CAD 106,178.35

106,178.35 – 100,000 = 6,178.35 Interest = CAD 6,178.35

**Note:** More frequent compounding generates more interest. The interest rate (i) is calculated for the monthly compounding period by dividing the APR by 12 and the number of compounding periods is 12 per year times three years.

**Problem: Future Value at Sproule**

1. (10,000.00) (1 + .0610)3 = 11,943.90 FV = CAD 11,943.90

**Note:** Sproule will receive the initial loan principal of CAD 10,000 plus interest in three years.

**Compound Interest**

11,943.90 – 10,000.00 = 1,943.90 Interest = CAD 1,943.90

**Note:** The original amount of the loan is deducted from the future value calculated in Part 1 to determine the interest earned.

**Simple Interest**

(10,000) (.0610) (3) = 1,830.00 Interest = CAD 1,830.00

**Note:** The initial principal is multiplied by the annual interest rate and the number of years in the loan to give the simple interest. This can be done because there is no compounding or interest on interest at the end of each year.

**Additional Interest due to Compounding**

1,943.90 – 1,830.00 = 113.90 Additional interest = CAD 113.90

**Note:** This is the difference between the compound and simple interest. The table below provides an additional perspective on how the amounts were determined.

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Simple Interest** | **Compound Interest** | **Interest Due to Compounding** |
| **Beginning Principal (CAD)** | **Interest****i = 6.10%** | **Beginning Principal (CAD)** | **Interest****i = 6.10%** |
| 1 | 10,000.00 | 610.00 | 10,000.00 | 610.00 | 0.00 |
| 2 | 10,000.00 | 610.00 | 10,610.00 | 647.21 | 37.21 |
| 3 | 10,000.00 | 610.00 | 11.257.21 | 686.69 | 76.69 |
|  | Total interest | 1,830.00 | Total interest | 1,943.90 | 113.90 |

1. (10,000.00) (1 + )(3 x 2) = 11,975.34 FV = CAD 11,975.34

**Note:** The interest earned grows with more frequent compounding.

**Problem:** **APR versus EAR at Tyson**

1.

8%, compounded monthly

(1 +)12 – 1 = .0830 or 8.30%

**Note:** The APR is divided by the number of compounding periods per year to determine the effective interest rate for the period. This rate is then compounded by the number of compounding periods per year to give the EAR.

8%, compounded quarterly

(1 +)4 – 1 = .0824 or 8.24%

8%, compounded semi-annually

(1 +)2 – 1 = .0816 or 8.16%

8%, compounded annually

(1 +)1 – 1 = .0800 or 8.00%

**Note:** The EAR is higher the more frequent the compounding.

**Problem: Present Value at Tribeca**

1. PV = CAD 139,289.91

**Note:** An amount equal to 150,000 must be accumulated in three years. If Tribeca invests 139,289.91 and earns interest for three years, it will have the 150,000 needed.

1.

1.25 = 1.025n

log 1.25 = n log 1.025

n =

n = 9.04 years

**Note:** It will take longer to accumulate 150,000 because at the same interest rate because you have less to invest now. Logarithms is a complex topic that was first introduced in high school. This is the only application you need to understand for this module.

1. i = .0772 or 7.72%

**Note:** A higher interest rate is required if Tribeca only has 120,000 to investment and must accumulate 150,000 in three years. The interest rate (i) is solved for by:

 (1 + i)

 i

i = .0772

**Problem: Present Value at Sol**

1. Yes, they should investment. The total present value of the future cash inflows at the end of each of the three years and the sale of the assets at the end of the project’s life exceeds the initial cost, so the project makes a profit. By expressing all cash flows in today’s dollars by calculating their PV, the cash flows are comparable.

|  |  |  |  |
| --- | --- | --- | --- |
| **Time** | **Cash Flows (CAD)** | **Calculation****i = 7%, compounded annually** | **Present Value (CAD)** |
| Initial investments | -75,000 |  | -75,000.00 |
| Cash inflows at end of year 1 | 45,000 |  | 42,056.07 |
| Cash inflows at end of year 2 | 45,000 |  | 39,304.74 |
| Cash inflows at end of year 3 | 45,000 |  | 36,733.40 |
| Sale of assets | 25,000 |  | 20,407.45 |
| Total | CAD 63,501.66  |

or

 + +-75,000 + 118,094.22 + 20,407.45 = 63,501.67

PV = CAD 63,501.67

**Note:** In the second component of the above formula, the PVA formula is used instead of finding the PV of each cash flow individually. Otherwise, the calculations are the same.

**Problem: Future Value of an Annuity at Cartlidge**

1.

Option 1

50,000 (1 + .0355)25 = 119,597.63 FV = CAD 119,597.63

Option 2

() = 137,234.77 FVA = CAD 137,234.77

Option 2 is preferred because the future value is higher.

**Note:** The FVA formula was used to value Option 2 because Cartlidge would be making equal payments at the end of each year.

1. No. The future value of Option 2 would be even higher as the investments are made at the beginning of the year and there is an extra year of compounding.

() (1 + .0355) = 142,106.60 FVAD = CAD 142,106.60

**Note:** The FVAD formula is used instead of the FVA formula because the payments are made at the beginning of the year.

3. It will discipline Cartlidge to invest her inheritance now. Other purchases or expenses in the future may prevent her from making regular payments over the next 25 years under Option 2.

**Problem: Present Value of an Annuity at Edwards**

1.

Option 1

 + = 748,745.64 PV = CAD 748,745.64

**Note:** The PV of the two payments must be calculated separately because they are not in successive years.

Option 2

 = 729,980.62 PVA = CAD 729,980.62

**Note:** The PVA formula can be used because the payments are in successive years.

Option 1 has a higher present value.

2.

Option 2

 = 759,179.84 PVAD = CAD 759,179.84

or

 = 759,179.84 PVAD = CAD 759,179.84

**Note:** Receiving the payments at the beginning of the year makes them more valuable.

Yes. Option 2 now has a higher present value.

**Problem: Present Value of an Annuity at Wellington**

1.

Bid 1

 + = 91,276.31 PV = CAD 91,276.31

**Note:** The PV of the first and second components of the formula must be determined separately because the amounts are different. The third component of the formula finds the PV of the payments in Years 3 through 10. The PVA formula is used because the amounts are the same each year. The PVA formula provides the value of the annuity at the beginning of Year 3 which is the same as the end of Year 2. All PV formulas provide a value at the beginning of the year. This amount needs to be discounted for two more years to determine its present value today.

Bid 2

 PVAD = CAD 81,436.48

or

 = 81,436.48 PVAD = CAD 81,436.48

**Note:** The PVAD formula is used because payments are at the beginning of the year.

Bid 1 is preferred.

**Problem: Present Value of an Annuity at Wilson**

1.

Option 1

 = 85,947.86 PV = CAD 85,947.86

Option 2

 = 92,660.82 PVA = CAD 92,660.82

Option 3

 + 42 PV = 86,013.42

 Option 2 is preferred as it has the highest present value.

**Problem: Present Value of an Annuity at Harte**

1.

Offer 1: Innovative Products

= + PV = CAD 157,781.68

Offer 2: Morden Industries

PV = CAD 145,000

Offer 1 is preferred as it has the highest present value.

**Problem: Present Value of a Perpetuity at Wexler**

1.

 = 104.00 PVP= CAD 104.00

**Note:** This is the present value of a perpetuity formula. The inputs were modified to give a quarterly payment (÷ 4) and a quarterly interest rate (÷ 4) but notice that it did not matter as the 4’s cancelled out. Remember a perpetuity is a string of payments that goes on forever and is commonly used in valuing common and preferred shares.

**Problem: Present Value of a Perpetuity with Growth at Jenkins**

1.

 = 267.80 PVPG = CAD 267.80

**Note:** This is the present value of a perpetuity with growth formula. The dividend is currently CAD 5.20 paid out annually. By the end of the first year it will be 3.0% higher with growth hence (5.20)(1 + .03). Deducting .03 in the denominator allows for growth in the numerator over time.

2.

 + + + = 5.35 + 5.50 + 5.66 + 291.42 = 307.93

PV = CAD 307.93

**Note:** The growth rate in the first three years was different than the subsequent years, so the present values of the first three payments were calculated separately. The PVPG formula was used for all remaining payments that go on forever with growth. This value is at the beginning of Year 4 or end of Year 3, so the amount was discounted back another three years to today’s value.

**Problem: Present Value of an Annuity with Growth at Harrison**

1. Yes, Harrison should undertake the project because the PV of the future cash inflows exceeds the initial cost.

-250,000 + ( (1 – ) = 50,873.12 PVAG = CAD 50,873.12

**Note:** This is the present value of an annuity with growth formula. It is used when the cash flows are expected to increase over the life of a project which is normal. The initial investment is not discounted because it occurs immediately.

**Problem: Calculating a Loan Payment at Jones**

1.

(450,000) (1 - .20) = P ()

Payment = 2,277.54

**Note:** Jones had to make a 20% down payment, so the loan is only for 80% of 450,000. Blended, equal monthly loan payments are an annuity since the same payment occurs at the end of each month for the life of the loan so the PVA formula is used. When you take the PV of loan payments, it removes the interest from the payments leaving only the principal which must total to the value of the loan. The interest rate and compounding period must be for one month since the payments are monthly. The number of periods is the number of payments or months in the loan.

2.

|  |
| --- |
| **Amortization Table** |
| **Period** | **Beginning Principal** | **Interest**(.045 ÷ 12) | **Principal** | **Ending Principal** |
| 1 | 360,000.001 | 1,350.002 | 927.543 | 359,072.464 |
| 2 | 359,072.465 | 1,346.526 | 931.027 | 358,141.448 |

1 (450,000) (1 - .20)

2 (360,000) (.045 / 12)

3 2,277.54 – 1,350.00

4 360,000 – 927.54

5 Ending principal of the period before

6 (359,072.46) ( .045 / 12)

7 2,277.54 – 1,346.52 = 931.02

8 359,072.46 – 931.02 = 358,141.44

**Note:** An amortization table provides the break down of interest and principal in loan payments over the life of the loan. Interest falls as the loan is paid down and principal rises as more of the payment is devoted to paying down the loan. The ending principal of one period is the beginning principal of the next period. By the end of the loan’s life in 240 months, the ending principal will be zero.

**Problem: Interest Rate at Wilson**

1.

250,000 = 1,700 ()

i = .005481 (solved using Goal Seek in Excel)

**Note:** Analysts frequently know the loan’s principal and the amount and number of payments, but do not know what interest rate (i) is implied by these amounts. The same formula can be used, analysts simply solve for a different unknown.

See the Excel spreadsheet Answer Keys – Interest Rate at Wilson Company to see how Goal Seek was used in this question.

**APR**

(.005481) (12) = .065772

6.58%, compounded monthly

**Note:** APR expresses the interest rate annually without the effect of compounding, so the monthly effective interest rate is multiplied by twelve months.

**EAR**

(1 + .005481)12 – 1 = .067791

6.78%, compounded annually

**Note:** EAR expresses the interest rate annually including the effect of compounding, so the monthly effective rate must be compounded 12 times. This formula can be expressed as investing CAD 1 and earning interest on a compounded basis for twelve months and then paying back the CAD 1 leaving just the interest. Because CAD 1 or 100 cents was used, the interest rate can be quickly converted to per cent form as the base is 100.

**Problem: Number of Payments at Allison**

1.

350,000 = 1,500 ()

350,000 = 1,500 ()

.680633 =

1.002917-n =.319367

-n log 1.002917 = log .319367

n = - = 391.87 months

**Note:** Logarithms must be used solve for an unknown when it is an exponent.

**Problem: Customized Loan Schedule at Hastings**

1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Interest Paid** | **Principal Paid** | **Total****Payment** | **Ending Principal** |
| September 30, 20181 | 110,000 | 0 | 110,000 | 5,500,000 |
| December 31, 20181 | 110,000 | 0 | 110,000 | 5,500,000 |
| March 31, 20191 | 110,000 | 500,000 | 610,000 | 5,000,000 |
| June 30, 20192 | 100,000 | 500,000 | 600,000 | 4,500,000 |
| September 30, 20193 | 90,000 | 500,000 | 590,000 | 4,000,000 |
| December 31, 20194 | 80,000 | 500,000 | 580,000 | 3,500,000 |
| March 31, 20205 | 70,000 | 750,000 | 820,000 | 2,750,000 |
| June 30, 20206 | 55,000 | 750,000 | 805,000 | 2,000,000 |
| September 30, 20207 | 40,000 | 750,000 | 790,000 | 1,250,000 |
| December 31, 2020,8 | 25,000 | 1,250,0009 | 1,275,000 | 0 |

1 (5,500,000) (.08/4) = 110,000

2 (5,500,000 – 500,000 (1)) (.08/4) = 100,000

3 (5,500,000 – 500,000 (2)) (.08/4) = 90,000

4 (5,500,000 – 500,000 (3)) (.08/4) = 80,000

5 (5,500,000 – 500,000 (4)) (.08/4) = 70,000

6 (5,500,000 – 500,000 (4) − 750,000 (1)) (.08/4) = 55,000

7 (5,500,000 – 500,000 (4) – 750,000 (2)) (.08/4) = 40,000

8 (5,500,000 – 500,000 (4) – 750,000 (3)) (.08/4) = 25,000

9 (5,500,000 – 500,000 (4) – 750,000 (3))

2.

I/O – Only interest was paid during the first two quarters.

Straight-line – An equal amount of principal was paid in 2019 and 2020.

Stepped – Principal payment increased from CAD 0 in 2018 to CAD 500,000 in 2019 to CAD 750,000 in 2020.

Balloon – Principal payments were deferred, and a large principal payment was made in the final quarter.

**Note:** Customized loan repayment schedules may be negotiated that result in lower total payments initially when a new company or project is generating a limited amount of cash. As the company or project becomes more successful, the principal payments increase, and the loan is eventually paid off.

**Problem: Time Value of Money Applications**

 = 39,672.88

PVP = = 137,500.00

(8) (5,500) (.038) = 1,672.00

|  |  |
| --- | --- |
| FVAD = | 3,500 ((1 + .0325) = 23,522.43 |

 = 41,551.27

|  |  |
| --- | --- |
| PVPG =  |  = 87,000.00 |

|  |  |
| --- | --- |
| PVA = | 4,200 () = 18,882.93 |

EAR = = .0356 or 3.56%

- 1 = .0614 or 6.14%

**Problem: Predefined FV and PV Functions in Excel**

1. See Excel spreadsheet provided.

**Problem: Calculating a Loan Payment at Flynn**

1. See Excel spreadsheet provided.
2. See Excel spreadsheet provided.