**Bond Valuation and Interest Rates**

**Learning Problems**

**Answer Keys**

**Valuing a Straight Bond at Rex**

1.

P0 = 112,000 ($\frac{1-(1+(\frac{.11}{2}))^{-20}}{\left(\frac{.11}{2}\right)}$) + $\frac{200,000}{(1+(\frac{.11}{2}))^{20}}$ = 211,950.38

1(1,000) (200) ($\frac{.12}{2}$)

= $\frac{211,950.38}{200,000.00}$ = 1.05975 or 105.975 (premium)

2.

P0 = 112,000 ($\frac{1-(1+(\frac{.13}{2}))^{-20}}{(\frac{.13}{2})}$) + $\frac{200,000}{(1+(\frac{.13}{2}))^{20}}$ = 188,981.49

1(1,000) (200) ($\frac{.12}{2}$)

= $\frac{188,981.49}{200,000.00}$ = .94491 or 94.491 (discount)

**Valuing a Straight Bond at SureFire**

1.

P0 = 11,000,000 ($\frac{1-(1+\left(\frac{.075}{2}\right))^{-30}}{(\frac{.075}{2})}$) + $\frac{25,000,000}{(1+(\frac{.075}{2}))^{30}}$ = 26,114,327.82

1(25,000) (1,000) ($\frac{.08}{2}$)

2.

= $\frac{26,114,327.82}{25,000,000}$ = 1.04457 or 104.457 (premium)

**Valuing a Straight Bond at ABC**

1.

P0 = 12,500 ($\frac{1-(1+(\frac{.12}{2}))^{-16}}{(\frac{.12}{2})}$) + $\frac{50,000}{(1+(\frac{.12}{2}))^{16}}$ = 44,947.05

1(50,000) ($\frac{.10}{2}$)

Note: Initially this was a 10-year bond, but it was purchased two years into its term. Eight years with two semi-annual payments per year equals 16 payments.

2.

= $\frac{44,947.05}{50,000}$ = .898941 or 89.894 (discount)

3. A forecast of falling interest rates means the value of a bond portfolio will rise if the forecast is correct. To maximize the increase, Higgins could:

* Increase the average term of the bond portfolio
* Increase the proportion of zero-coupon bonds in the portfolio

**Valuing a Straight Bond at Elford**

1. **7% Market Rate - Bond Quotation**

P0 = 118,000 ($\frac{1-(1+(\frac{.07}{2}))^{-32}}{(\frac{.07}{2})}$) + $\frac{450,000}{(1+(\frac{.07}{2}))^{32}}$ = 492,904.95

1(450,000) ($\frac{.08}{2}$)

= $\frac{492,904.95}{450,000}$ = 1.09534 or 109.534 (premium)

**8% Market Rate – Bond Quotation**

P0 = 118,000 ($\frac{1-(1+(\frac{.08}{2}))^{-32}}{(\frac{.08}{2})}$) + $\frac{450,000}{(1+(\frac{.08}{2}))^{32}}$ = 450,000.00

1(450,000) ($\frac{.08}{2}$)

= $\frac{450,000}{450,000}$ = 1.00000 or 100.000 (par)

Note: The calculation for the 8.0% market rate is not required. If the market rate and coupon rate of the bond are the same, the will bond will trade at par.

1. If the bond is currently trading at 98.000, Global Bond Ltd. should buy the bond as they expect it to trade in the 100 to 109.534 range based on their interest rate forecast.

**Zero-Coupon Bond at Williams**

1. P0 = $\frac{200,000}{(1+(\frac{.08}{2}))^{30}}$ = CAD 61,663.73
2. P0 = $\frac{200,000}{(1+(\frac{.08}{2}))^{28}}$ = CAD 66,695.50
3. Even though no interest is paid, for tax purposes, the increase in the value of the zero-coupon bonds is treated as interest income and taxed at normal rates. It is not treated as a capital gain that is only taxed at 50% of the normal tax rate.

**Effect of Term and Cash Flow Pattern on Interest Rate Risk at Henson**

1.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Bond 1** | **Bond 2** | **Bond 3** |
| Before change | 150,000.00 | 150,000.00 | 62,196.43 |
| After change | 2144,208.70 | 3140,653.34 | 456,533.42 |
| Percentage change | -3.86% | -6.23% | -9.11% |

1 $= \frac{150,000}{(1+(\frac{.09}{2}))^{20}}$

2 = (150,000)($\frac{.09}{2}$)($\frac{1-(1+(\frac{.10}{2}))^{-10}}{(\frac{.10}{2})}$) + $\frac{150,000}{(1+(\frac{.10}{2}))^{10}}$

3 = (150,000)($\frac{.09}{2}$)($\frac{1-(1+(\frac{.10}{2}))^{-20}}{(\frac{.10}{2})}$) + $\frac{150,000}{(1+(\frac{.10}{2}))^{20}}$

4 = $\frac{150,000}{(1+(\frac{.10}{2}))^{20}}$

Rising interest rates cause bond prices to fall, but Bond 3 will fall the most because:

* Bond 3 and Bond 2 have the longest term at 10 years, but
* Bond 3 has the most deferred cash flows as it is a zero-coupon bond

**Re-investment Risk at Smithson**

1. Yield to maturity is 10.0%, compounded semi-annually which is the market rate.
2. No, if the interest paid every six months is reinvested at the coupon rate, then the yield to maturity will remain at 10.0%, compounded semi-annually.

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Principal****+****Interest** | **FV Factor** | **FV** |
| 1 | 2,500 | (1+ .05)9 | 3,878.32 |
| 2 | 2,500 | (1+ .05)8 | 3,693.64 |
| 3 | 2,500 | (1+ .05)7 | 3,517.75 |
| 4 | 2,500 | (1+ .05)6 | 3,350.24 |
| 5 | 2,500 | (1+ .05)5 | 3,190.70 |
| 6 | 2,500 | (1+ .05)4 | 3,038.77 |
| 7 | 2,500 | (1+ .05)3 | 2,894.06 |
| 8 | 2,500 | (1+ .05)2 | 2,756.25 |
| 9 | 2,500 | (1+ .05)1 | 2,625.00 |
| 10 | 52,500 | (1+ .05)0 | 52,500.00 |
| 81,444.73 |

50,000 (1 + i)10 = 81,444.73

i = .05

(.05) (2) = .10 or 10.0%

Yield to maturity is 10.0%, compounded semi-annually

1. Yes, if the interest paid every six months is reinvested at a rate below the coupon rate, then the yield to maturity will decrease. This is reinvestment risk.

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Principal****+****Interest** | **FV Factor** | **FV** |
| 1 | 2,500 | (1+ .025)9 | 3,122.15 |
| 2 | 2,500 | (1+ .025)8 | 3,046.01 |
| 3 | 2,500 | (1+ .025)7 | 2,971.71 |
| 4 | 2,500 | (1+ .025)6 | 2,899.23 |
| 5 | 2,500 | (1+ .025)5 | 2,828.52 |
| 6 | 2,500 | (1+ .025)4 | 2,759.53 |
| 7 | 2,500 | (1+ .025)3 | 2,692.23 |
| 8 | 2,500 | (1+ .025)2 | 2,626.56 |
| 9 | 2,500 | (1+ .025)1 | 2,562.50 |
| 10 | 52,500 | (1+ .025)0 | 52,500.00 |
| 78,008.44 |

50,000 (1 + i)10 = 78,008.44

i = .0455

(.0455) (2) = .0910 or 9.10%

Yield to maturity is 9.10%, compounded semi-annually

**Yield Curve for A-Rated Companies**

1.

January 1, 2010 – Normal upward sloping yield curve that indicates rising rates

January 1, 2015 – Inverted yield curve that indicates falling rates

2.

 January 1, 2010

 1r2 (1 + .0444)1 (1 + x)2 = (1+.0525)3  x = .0566 or 5.66%

 2r1 (1 + .0498)2 (1 + x)1 = (1 + .0525)3  x = .0579 or 5.79%

 3r3 (1 + .0525)3 (1 + x)3 = (1 + .0603)6  x = .0682 or 6.82%

 January 1, 2015

 1r2 (1 + .0845)1 (1 + x)2 = (1+.0632)3  x = .0527 or 5.27%

 2r1 (1 + .0745)2 (1 + x)1 = (1 + .0632)3  x = .0410 or 4.10%

 3r3 (1 + .0632)3 (1 + x)3 = (1 + .0484)6  x = .0338 or 3.38%

3.

 January 1, 2010

(1 + .0444) (1 + .0566)2 (1 + .0682)3 = (1 +.0603)6

 1.42 = 1.42

 January 1, 2015

 (1 + .0845) (1 + .0527)2 (1 + .0338)3 = (1 +.0484)6

 1.33 = 1.33

**Yield Curve for the Food Processing Industry**

1.

 3r2 (1 + .0523)5 =(1 + .0462)3 (1 + 3r2)2  3r2 = .0615 or 6.15%

 2r3 (1 + .0523)5 = (1 + .0458)2 (1 + 2r3)3 2r3 = .0567 or 5.67%